COMBINATORICS AT THE CONFLUENCE

Abstract Booklet

July 20-22, 2026
Carnegie Mellon University
University of Pittsburgh
Pittsburgh, Pennsylvania

Gates Hillman Center
Rashid Auditorium (GHC 4401) · GHC 4307 · GHC 4405

Program at a glance

All plenary talks and the ICARM tutorial take place in Rashid Auditorium, GHC 4401. Parallel-session room assignments are listed on the following page.

Monday, July 20

Time	Event	Location
9:00-9:30	Registration	Gates Hillman Center
9:30-10:30	**Pavel Galashin** - Amplituhedra and origami	Rashid, GHC 4401
10:30-11:00	Coffee break	Gates Hillman Center
11:00-12:30	Parallel special sessions	GHC 4401 / 4307 / 4405
12:30-2:00	Break	
2:00-3:00	**Mihyun Kang** - Matchings in sparse random graphs	Rashid, GHC 4401
3:00-3:30	Coffee break	Gates Hillman Center
3:30-4:30	Parallel special sessions	GHC 4401 / 4307 / 4405
4:30-5:00	Coffee break	Gates Hillman Center
5:00-6:00	**Adam Wagner** - Exploring constructions in mathematics using AI tools (ICARM)	Rashid, GHC 4401
6:00-8:00	Poster session and reception	Rangos Ballroom, Cohon University Center

Tuesday, July 21

Time	Event	Location
9:30-10:30	**Adam Wagner** - AlphaEvolve tutorial (ICARM)	Rashid, GHC 4401
10:30-11:00	Coffee break	Gates Hillman Center
11:00-12:30	Parallel special sessions	GHC 4401 / 4307 / 4405
12:30-2:00	Break	
2:00-3:00	**Lauren Williams** - Combinatorics of Macdonald polynomials and their generalizations	Rashid, GHC 4401
3:00-3:30	Coffee break	Gates Hillman Center
3:30-4:30	Parallel special sessions	GHC 4401 / 4307 / 4405
4:30-5:00	Coffee break	Gates Hillman Center
5:00-6:00	**Rob Morris** - Recent results in Ramsey theory	Rashid, GHC 4401

Wednesday, July 22

Time	Event	Location
9:30-10:30	**Hailun Zheng** - Rigidity, stress spaces, and lower bound problems on simplicial spheres	Rashid, GHC 4401
10:30-11:00	Coffee break	Gates Hillman Center
11:00-12:00	**Petter Brändén** - Totally nonnegative matrices and zeros of poset polynomials	Rashid, GHC 4401
12:00	Conference concludes	

Parallel-session schedule

Talks are 25 minutes, followed by a 5-minute interval for movement between rooms.

Day	Time	Extremal Rashid, GHC 4401	Probabilistic GHC 4307	Algebraic / Geometric / Topological GHC 4405
Mon	11:00-11:25	**Alex Scott** Some problems in coarse graph theory	**Jane Gao** Satisfiability of random CSPs: Uniquely extendable and non-uniquely extendable	**Sarah Brauner** q-deformations of shuffling operators
Mon	11:30-11:55	**Richard Montgomery** Nearly-uniform degree distributions in spanning subgraphs	**Felix Joos** Fractional clique decompositions of the binomial random graph	**Martha Yip** Flag positroid pipe dreams
Mon	12:00-12:25	**Anton Bernshteyn** Probabilistic combinatorics and dynamical systems	**Florian Pfender** Improving R(3,k) in just two bites	**Tyrrell McAllister** Boundaries of pseudo-integral polygons
Mon	3:30-3:55	**Michelle Delcourt** On Nash-Williams' Conjecture	**Tom Kelly** How far can the Rödl nibble go?	**Hunter Spink** The quasisymmetric flag variety
Mon	4:00-4:25	**Luke Postle** A Proof of Nash-Williams' Conjecture	**Jeff Kahn** On thresholds for spanning trees	**Shira Zerbib** The chromatic number of 3-stable Kneser graphs
Tue	11:00-11:25	**Liana Yepremyan** Finding a tree in the jungle	**Wes Pegden** Random volumes in d-dimensional polytopes	**Daoji Huang** Dual affine Robinson-Schensted correspondence via growth diagrams
Tue	11:30-11:55	**Adva Mond** Maker-Breaker percolation games on a random board	**Michael Molloy** Short cycles in the d-process	**Patricia Hersh** A new ribbon basis for rank-selected homology of geometric lattices and representation stability
Tue	12:00-12:25	**Wojciech Samotij** Sparse sets in hypergraphs	**Fan Chung** Eigenvalues of clustering graphs	**Gaku Liu** The Gröbner version of White's conjecture is false
Tue	3:30-3:55	**Sam Mattheus** The limits of the inertia bound for graph powers	**Cosmin Pohoata** Sidon sets in the squares, repeated distances, and the Elekes-Rónyai problem	**Lorenzo Vecchi** Existence of Kähler algebras with Chow polynomials as Hilbert series
Tue	4:00-4:25	**Julian Sahasrabudhe** On the Erdős-Rogers function	**Simon Griffiths** Permutation robust random graph thresholds	**Josephine Yu** Principal minors of positive-definite matrices

Plenary Talks and ICARM Tutorial

Rashid Auditorium, GHC 4401 · Monday-Wednesday

Monday, July 20 · 9:30 a.m. · Rashid Auditorium, GHC 4401

Amplituhedra and origami

Pavel Galashin

ABSTRACT

I will outline the combinatorial machinery behind a recent proof of the BCFW triangulation conjecture for the amplituhedron. I will mostly focus on the combinatorics of the dimer model on bipartite graphs embedded in a disk, including Kenyon-Smirnov primitives, origami crease patterns, and two punctured versions of planar bipartite graphs related by a surprising combinatorial duality.

Monday, July 20 · 2:00 p.m. · Rashid Auditorium, GHC 4401

Matchings in sparse random graphs

Mihyun Kang

ABSTRACT

We begin with the classical result of Karp and Sipser on maximum matchings in sparse Erdős-Rényi random graphs. Building on this foundation, we present recent progress: a central limit theorem for the matching number, a local limit theorem for the Karp-Sipser core, and the universal behavior of the rescaled matching number of sparse random graphs. We conclude with several open problems and directions for future research.

Monday, July 20 · 5:00 p.m. · Rashid Auditorium, GHC 4401

Exploring constructions in mathematics using AI tools

Adam Wagner

ABSTRACT

In mathematics, the focus is often on finding interesting constructions for various mathematical problems. I will discuss how some tools can reduce the effort required to find the right questions, explore variants of a problem, test our ideas, and find good constructions for various problems. We will look at different ways we can think about how these tools work, examine many examples, and determine which of these setups is right for the problems you care about.

Tuesday, July 21 · 9:30 a.m. · Rashid Auditorium, GHC 4401

AlphaEvolve tutorial (ICARM)

Adam Wagner

Tuesday, July 21 · 2:00 p.m. · Rashid Auditorium, GHC 4401

Combinatorics of Macdonald polynomials and their generalizations

Lauren Williams

ABSTRACT

Macdonald polynomials are a famous family of symmetric polynomials connected to the Hilbert scheme, knot invariants, and the Hecke algebra. There are several combinatorial formulas for Macdonald polynomials, including the 2004 tableaux formula of Haglund-Haiman-Loehr and the 2018 multiline queue formula of Corteel-Mandelshtam-Williams.

In 1996, Knop and Sahi introduced a remarkable family of inhomogeneous symmetric polynomials called interpolation Macdonald polynomials, which are defined via vanishing conditions and generalize Macdonald polynomials in the sense that their top homogeneous parts are exactly the Macdonald polynomials.

In recent work with Houcine Ben Dali, we give the first combinatorial formula for interpolation Macdonald polynomials; our formula is in terms of signed multiline queues and generalizes the multiline queue formula for Macdonald polynomials. We also give a tableaux formula. If time permits, I will discuss how these polynomials are connected to interacting particle systems.

Tuesday, July 21 · 5:00 p.m. · Rashid Auditorium, GHC 4401

Recent results in Ramsey theory

Rob Morris

ABSTRACT

Over the past few years there has been a series of remarkable breakthroughs in Ramsey theory. In this talk we will discuss several of these, including an exponential improvement for the diagonal Ramsey numbers, stunning new constructions for off-diagonal Ramsey numbers, and an exponential upper bound for induced Ramsey numbers.

Wednesday, July 22 · 9:30 a.m. · Rashid Auditorium, GHC 4401

Rigidity, stress spaces, and lower bound problems on simplicial spheres

Hailun Zheng

ABSTRACT

The Generalized Lower Bound Theorem conjectured by McMullen and Walkup in the early 1970s and proved through work of Stanley, Kalai, Murai-Nevo, and Adiprasito asserts that the g_i-numbers of any simplicial (d - 1)-sphere are nonnegative, and that for each i, the minimum $g_i = 0$ is achieved precisely by (i - 1)-stacked spheres. Kalai's proof of the Lower Bound Theorem (the i = 2 case) via rigidity theory established a paradigm central to the field.

A natural refinement asks for sharp lower bounds on g_i within the class S(j,d - 1) of simplicial (d - 1)-spheres whose missing faces all have dimension at most j; for most pairs (i,j) these bounds remain open, including the Charney-Davis conjecture for S(1,d - 1) - the class of flag (d - 1)-spheres.

In this talk, I will discuss recent progress on lower bound problems for spheres in S(j,d - 1). The proofs blend Stanley-Reisner ring theory, affine stress spaces, and rigidity theory. Results include new bounds on the g-numbers of spheres in certain classes S(j,d - 1) - including S(1,d - 1) and S(2,4) - as well as a classification of S(3,5)-spheres with $g_3 = 1$.

This is joint work with Isabella Novik.

Wednesday, July 22 · 11:00 a.m. · Rashid Auditorium, GHC 4401

Totally nonnegative matrices and zeros of poset polynomials

Petter Brändén

ABSTRACT

We associate several families of univariate polynomials to any unitriangular matrix (a lower triangular matrix with all diagonal entries equal to one). When the matrix is totally nonnegative, the polynomials are proven to be real-rooted. When the matrix comes from a partially ordered set (poset), these polynomials are known as chain polynomials and Chow polynomials. In this setting our results prove new cases of conjectures regarding real-rootedness of such polynomials.

This is based on joint work with Leonardo Saud Maia Leite and Lorenzo Vecchi.

Extremal Combinatorics

Rashid Auditorium, GHC 4401 · Monday-Tuesday

Monday, July 20 · 11:00 a.m. · Rashid Auditorium, GHC 4401

Some problems in coarse graph theory

Alex Scott

ABSTRACT

Coarse graph theory is a developing area that focuses on the large-scale geometric structure of graphs, particularly through the lens of quasi-isometry. A central goal is to find coarse analogues of classical graph-theoretic results. We discuss some current progress in this direction.

Joint work with Tung Nguyen and Paul Seymour.

Monday, July 20 · 11:30 a.m. · Rashid Auditorium, GHC 4401

Nearly-uniform degree distributions in spanning subgraphs

Richard Montgomery

ABSTRACT

How irregular can the degree distribution of a spanning subgraph of a d-regular n-vertex graph be? As there are d + 1 possible vertex degrees in such a subgraph, the best we could hope for is to have around n/(d + 1) vertices of each degree in {0,1,...,d}. I will discuss a result showing that if d = o(n), then there is always a subgraph in which there are (1 + o(1))n/(d + 1) vertices of degree i, for each i ∈ {0,1,...,d}. This proves a conjecture of Alon and Wei and strengthens a previous result of Fox, Luo, and Pham.

This is joint work with Alexey Pokrovskiy and Benny Sudakov.

Monday, July 20 · 12:00 p.m. · Rashid Auditorium, GHC 4401

Probabilistic combinatorics and dynamical systems

Anton Bernshteyn

ABSTRACT

In recent years, tools from combinatorics, especially the probabilistic method, have found unexpected applications in the field of dynamical systems theory. In this talk, I will discuss this research trend by focusing on one recent example, namely the construction of flows with minimal subdynamics for arbitrary countable groups.

Based on joint work with Joshua Frisch.

Monday, July 20 · 3:30 p.m. · Rashid Auditorium, GHC 4401

On Nash-Williams' Conjecture

Michelle Delcourt

ABSTRACT

A central open question in extremal design theory is Nash-Williams' Conjecture from 1970, namely that every triangle-divisible graph on n vertices (for n large enough) with minimum degree at least 0.75n has a triangle decomposition. In this talk, we discuss the history of the problem and our recent resolution of this conjecture, as well as other applications in design theory.

This is joint work with Luke Postle.

Monday, July 20 · 4:00 p.m. · Rashid Auditorium, GHC 4401

A Proof of Nash-Williams' Conjecture

Luke Postle

ABSTRACT

In 1970, Nash-Williams conjectured that every triangle-divisible graph on n vertices with minimum degree at least 3n/4 has a triangle decomposition, provided n is large enough. In this talk, we overview our recent proof of this conjecture, highlighting the new techniques we developed to resolve the fractional version as well as the full conjecture.

Joint work with Michelle Delcourt.

Tuesday, July 21 · 11:00 a.m. · Rashid Auditorium, GHC 4401

Finding a tree in the jungle

Liana Yepremyan

ABSTRACT

We consider the problem of detecting and reconstructing trees planted in a sparse random graph. More specifically, given a k-vertex tree T, we consider the union of a random graph G(n,p), where $p = c/n$, and a copy of T randomly planted into the vertex set of the random graph. We study how large k has to be so that the presence of the planted tree can be detected. We distinguish two cases: (1) the shape of the tree T is known; and (2) T is an unknown uniformly random tree.

When T is known, we show that if $k = \Omega(\log n)$ and $c < 1.12$, then for most trees T, the planted model can be distinguished from G(n,c/n). This may be surprising since, for $c \geq 1$, G(n,c/n) contains random trees of size $n^{2/3-o(1)}$. On the other hand, when $p \geq C/n$ for a large constant C and $k = o(\sqrt{n})$, no test can detect the presence of the planted tree for most trees. Both bounds on k are optimal up to a constant factor. Even when the shape of the planted tree is unknown, $k = \omega(\log^2 n)$ is sufficient for detection when $c \leq 1.14$. We also consider the problem of reconstructing the planted unknown random tree and show that partial reconstruction is possible when $k = \omega(\log^2 n)$.

Joint work with Nicolas Broutin, Nina Kamčev, Gábor Lugosi, and Bruce Reed.

Tuesday, July 21 · 11:30 a.m. · Rashid Auditorium, GHC 4401

Maker-Breaker percolation games on a random board

Adva Mond

ABSTRACT

Let G be the percolated infinite square lattice with parameter p. Maker and Breaker then play the following game by alternately claiming m and b edges of G, respectively. Breaker's goal is to isolate the origin from infinity. Maker's goal is to indefinitely avoid Breaker's win, meaning that she wants to defend the origin in an infinite connected component after deleting Breaker's edges.

It is known that on the unpercolated square lattice, meaning when $p = 1$, Maker wins the game. Using tools from bootstrap percolation, we prove that whenever $p < 1$, Breaker wins, showing that Maker's win on the unpercolated board is rather fragile.

This is joint work with Vojtěch Dvořák and Victor Souza.

Tuesday, July 21 · 12:00 p.m. · Rashid Auditorium, GHC 4401

Sparse sets in hypergraphs

Wojciech Samotij

ABSTRACT

Suppose that H is a uniform hypergraph with vertex set V. The hypergraph container lemmas supply a decomposition of the family of subsets of V that are independent in H into a "small" number of "simple" pieces. We will present a similarly spirited decomposition lemma that employs a weaker notion of "simple" but applies to arbitrary families of subsets of V.

This is joint work with Gady Kozma, Asaf Shapira, and Adam Zsolt Wagner.

Tuesday, July 21 · 3:30 p.m. · Rashid Auditorium, GHC 4401

The limits of the inertia bound for graph powers

Sam Mattheus

ABSTRACT

The inertia bound is a classical tool from spectral graph theory for upper-bounding the independence number of graphs. However, until a few years ago, we did not have a single example where the inertia bound was provably not tight. Since then, several researchers have investigated its tightness. Most notably, a recent result of Kwan and Wigderson shows that in locally sparse graphs (i.e., C_4-free graphs or graphs of girth 5), the inertia bound is always linear in the number of vertices, while examples of such graphs are known-for example, from finite geometry-with much smaller independence number. Follow-up work of Tang, Zang, and Elphick showed even stronger separation results for complements of pseudorandom graphs, relying on known constructions of triangle-free pseudorandom graphs.

We investigate the tightness of the inertia bound for graph powers. The k-th power of a graph G in this setting is the graph $G^{(k)}$ on the same set of vertices, with two vertices adjacent if their distance in G is at most k. This class of graphs appears in several contexts, such as the theory of distance-regular graphs and coding theory. An important property is that these graphs are never locally sparse, nor are they easily seen to be complements of pseudorandom graphs, and hence they require new ideas. Using random Cayley graphs and extending results of Alon, we show that for graph powers there can also be a large gap between the inertia bound and the actual independence number.

Joint work with Aida Abiad and Nils van de Berg (TU Eindhoven).

Tuesday, July 21 · 4:00 p.m. · Rashid Auditorium, GHC 4401

On the Erdős-Rogers function

Julian Sahasrabudhe

ABSTRACT

In this talk, I will discuss some recent progress on a relative of the classical Ramsey problem introduced by Erdős and Rogers: What is the largest K_s-free subset that can be found in every K_{s+1}-free graph on n vertices?

This is based on joint work with Rob Morris and Jacques Verstraete.

Probabilistic Combinatorics

GHC 4307 · Monday-Tuesday

Monday, July 20 · 11:00 a.m. · GHC 4307

Satisfiability of random CSPs: Uniquely extendable and non-uniquely extendable

Jane Gao

ABSTRACT

Determining the satisfiability threshold lies at the center of research in random constraint satisfaction problems. When adding random constraints one at a time to a set of n variables, the system transitions from being satisfiable (SAT) to unsatisfiable (UNSAT) when the number of constraints is around some critical value αn. Famous random CSP instances include k-SAT, k-XORSAT, graph and hypergraph coloring and independent-set problems, and linear equations over fields. Some of these lie in the class of uniquely extendable CSPs, such as k-XORSAT and linear equations over fields, whereas k-SAT, graph coloring, and independent-set problems are extendable but not uniquely extendable.

It has been observed that the satisfiability thresholds α* of k-XORSAT and linear equations over finite fields coincide regardless of the order of the field. Moreover, unique extendability governs the solution geometry and motivated a conjecture of Connamacher and Molloy that the SAT threshold of a random uniquely extendable CSP coincides with α*. We explore this conjecture and give a partial confirmation. We then explore what happens for non-uniquely extendable CSPs, and in particular the SAT thresholds for equations over finite rings. Equations over rings differ from the above-mentioned CSPs in that they are not even extendable, providing a unique perspective on what governs the behavior of SAT thresholds.

This talk is based on joint work with Theodore Morrison.

Monday, July 20 · 11:30 a.m. · GHC 4307

Fractional clique decompositions of the binomial random graph

Felix Joos

ABSTRACT

Given a natural number r, let p_r be the threshold at which every edge in G(n,p) is contained in a copy of K_r. Clearly, p_r is a lower bound for the threshold at which G(n,p) admits a (fractional) K_r-decomposition. We present progress on constructing fractional K_r-decompositions of G(n,p) close to p_r.

This is joint work with Zak Smith.

Monday, July 20 · 12:00 p.m. · GHC 4307

Improving R(3,k) in just two bites

Florian Pfender

ABSTRACT

We present a random construction proving that the extreme off-diagonal Ramsey numbers satisfy $R(3,k) \geq (\frac{1}{2} + o(1))k^2/\log k$ (conjectured to be asymptotically tight), improving the previously best bound $R(3,k) \geq (\frac{1}{3} + o(1))k^2/\log k$. In contrast to all previous constructions achieving the correct order of magnitude, we do not use a nibble argument.

Beyond the paper, we will explore a bit further how the approach can be used for other problems.

This is joint work with Zion Hefty, Paul Horn, and Dylan King.

Monday, July 20 · 3:30 p.m. · GHC 4307

How far can the Rödl nibble go?

Tom Kelly

ABSTRACT

The Rödl nibble is a fundamental probabilistic approach for finding large matchings in hypergraphs. Classical results of Pippenger and Frankl-Rödl show that the nibble produces almost-perfect matchings in nearly D-regular hypergraphs when the maximum 2-degree is much smaller than D, and later work of Vu sharpened the quantitative dependence by incorporating higher codegrees. In this talk, I will discuss joint work with Stephen Gould that pushes this line further: we show that the nibble can essentially exhaust the full codegree sequence, up to natural bottlenecks, even when codegrees exhibit significant “clustering.” I will explain the main idea behind the improvement, some generalizations to matchings in “partite” settings, and some applications to Latin squares and designs.

Monday, July 20 · 4:00 p.m. · GHC 4307

On thresholds for spanning trees

Jeff Kahn

ABSTRACT

The (old) question considered here is, roughly: given D (possibly growing) and large n, for what p = p(n,D) is G(n,p) likely to contain a copy of any particular n-vertex tree with maximum degree at most D?

Here G(n,p) is the usual Erdős-Rényi random graph.

The question is settled (more or less, and not easily) for fixed D, but for growing D less is known, though the answer is not hard to guess. We remain unable to prove this guess is correct, but report some interesting progress.

Joint work with Caleb Fong and Jinyoung Park.

Tuesday, July 21 · 11:00 a.m. · GHC 4307

Random volumes in d-dimensional polytopes

Wes Pegden

ABSTRACT

Suppose we choose N points uniformly at random from a convex body in d dimensions. How large must N be, asymptotically with respect to d, so that the convex hull of the points is nearly as large as the convex body itself? It was shown by Dyer-Füredi-McDiarmid that exponentially many samples suffice when the convex body is the hypercube, and by Pivovarov that the Euclidean ball demands roughly $d^{d/2}$ samples. We show that when the convex body is the simplex, exponentially many samples suffice; this then implies the same result for any convex simplicial polytope with at most exponentially many faces.

Joint work with Alan Frieze and Tomasz Tkocz.

Tuesday, July 21 · 11:30 a.m. · GHC 4307

Short cycles in the d-process

Michael Molloy

ABSTRACT

We study a classic model for forming a random graph of maximum degree d. Begin with n vertices and repeatedly add an edge between a uniformly chosen pair of vertices satisfying: (i) both have degree less than d, and (ii) they are not already adjacent. Ruciński and Wormald analyzed the short-cycle distribution for the case d = 2. We study the case of constant d > 2. We show that for any fixed t > 2, the number of t-cycles has a Poisson distribution with constant mean. The proof combines a switching argument and a differential-equations argument.

Joint work with Lora Hreish and Lutz Warnke.

Tuesday, July 21 · 12:00 p.m. · GHC 4307

Eigenvalues of clustering graphs

Fan Chung

ABSTRACT

Many real-world networks possess the so-called small-world phenomenon, where every node is relatively close to every other node, and have a large clustering coefficient-that is, friends of friends are likely to be friends. We investigate the clustering effect in sparse clustering graphs by examining the eigenvalues, as well as quasirandom classes for strongly regular clustering graphs.

Tuesday, July 21 · 3:30 p.m. · GHC 4307

Sidon sets in the squares, repeated distances, and the Elekes-Rónyai problem

Cosmin Pohoata

ABSTRACT

We discuss a new combinatorial large-sieve method that uses algebraic splitting modulo many small primes to turn local congruence restrictions into global constraints on repeated values. This has various applications: (i) every Sidon subset of $\{1^2, 2^2, \ldots, N^2\}$ has size at most $N \cdot \exp(-c \log N/\log\log N)$, the first super-polylogarithmic saving for a classical problem of Alon and Erdős; (ii) a new upper bound on the largest subset of $[N]^2$ with no repeated distances, a problem of Erdős and Guy; and (iii) a new upper bound on the largest subset of $[N]^2$ with no isosceles triangle, a problem recently popularized by Charton, Ellenberg, Wagner, and Williamson.

This is based on recent joint work with Ernie Croot, Junzhe Mao, Adam Sheffer, and Kyle Yip. We will also discuss how these ideas recently led to a counterexample for the Elekes-Rónyai problem, and to a few other constructions.

Tuesday, July 21 · 4:00 p.m. · GHC 4307

Permutation robust random graph thresholds

Simon Griffiths

ABSTRACT

We consider the question of what structure can be guaranteed “robustly” in the intersection of two random graphs. For a graph property $\mathcal{P}$, one may ask for which values of (p,q) we have $\pi(G) \cap H \in \mathcal{P}$ for all bijective functions $\pi: V(G) \to V(H)$, where $G \sim G(n,p)$ and $H \sim G(n,q)$ are independent random graphs. We consider thresholds for properties such as containing an edge, containing a certain fixed subgraph, containing a giant component, and containing spanning subgraphs. Many open problems remain.

Based on joint work with Gautier Audhuy and Bruno Baldissera.

Algebraic, Geometric, and Topological Combinatorics

GHC 4405 · Monday-Tuesday

Monday, July 20 · 11:00 a.m. · GHC 4405

q-deformations of shuffling operators

Sarah Brauner

ABSTRACT

The k-random-to-random shuffle is a Markov chain that works as follows: one removes k cards from a deck at random and reinserts those k cards into new, uniformly random positions. To understand the convergence behavior of this process, one would like to know its eigenvalues-a surprisingly difficult question that remained open for more than a decade. It was resolved by Dieker-Saliola (k = 1) and Lafrenière (k > 1) in 2019, but the formulas for k > 1 were abstract enough that the second-largest eigenvalue was only conjectured.

In this talk, I will describe work with Commins, Grinberg, and Saliola that gives a q-deformation of this process to the type A Iwahori-Hecke algebra, where one may understand the parameter q as encoding a probability. We prove that the eigenvalues are always polynomials in q with nonnegative integer coefficients and can be expressed as certain evaluations of supersymmetric functions. This allows us to verify-and generalize-Lafrenière's conjecture for the second-largest eigenvalue.

Monday, July 20 · 11:30 a.m. · GHC 4405

Flag positroid pipe dreams

Martha Yip

ABSTRACT

We introduce flag positroid pipe dreams (FPPs), combinatorial objects that serve a similar purpose for flag positroids as Le-diagrams do for positroids. FPPs are in bijection with intervals in the Bruhat order and hence index Richardson cells in the decomposition of the nonnegative flag variety. We give a characterization of quotients of flag positroids with consecutive ranks in terms of partial FPPs, and given a full FPP, we can recover the constituents of a full flag positroid in terms of their Le-diagrams. We also address a problem of Chen et al. on a characterization of positroid quotients in terms of the cyclic-shift operators of Benedetti, Chavez, and Tamayo.

Monday, July 20 · 12:00 p.m. · GHC 4405

Boundaries of pseudo-integral polygons

Tyrrell McAllister

ABSTRACT

A polygon P is pseudo-integral if, as in the case of integral polygons, the Ehrhart function of P is a polynomial. Such triangles arise in algebraic geometry, where their duals determine fake weighted projective planes, and in symplectic geometry, where they reveal the “staircase” structure of the minimum radius of embeddings of certain symplectic ellipsoids.

We prove that a rational pseudo-integral triangle with exactly one lattice point in its interior has at most 9 lattice points on its boundary, as with integral triangles. We further show that such a triangle never has exactly 7 lattice points on its boundary. In addition, we construct convex pseudo-integral polygons with i interior lattice points and b boundary lattice points for all positive integral values of (i,b) such that $b \leq 5i + 4$. This contrasts with integral polygons, which must satisfy $b \leq 2i + 7$ by a result of Scott. Computations by Bohnert confirm that when the vertices of P are all in $\frac{1}{3}\mathbb{Z}^2$, our construction attains the maximum possible number of boundary lattice points for pseudo-integral polygons with $i \leq 5$ interior lattice points.

This is joint work with Jason Williford.

Monday, July 20 · 3:30 p.m. · GHC 4405

The quasisymmetric flag variety

Hunter Spink

ABSTRACT

Homology classes in the complete flag variety GL_n/B correspond to linear functionals on the ring of symmetric coinvariants. In this talk, I will describe recent work with Nantel Bergeron, Lucas Gagnon, Philippe Nadeau, and Vasu Tewari on how linear maps between flag varieties corresponding to “setting variables to zero” give rise to an unusually combinatorially Schubert-positive subvariety of the flag variety-the “quasisymmetric flag variety.”

Monday, July 20 · 4:00 p.m. · GHC 4405

The chromatic number of 3-stable Kneser graphs

Shira Zerbib

ABSTRACT

For an integer $s \geq 2$, a subset $S \subseteq [n]$ is s-stable if $\min\{j - i, n + i - j\} \geq s$ for every $i,j \in S$ with $i < j$. Denote the family of all s-stable subsets of size k of [n] by $C_s(n,k)$. Schrijver proved in 1978 that whenever $n \geq 2k$, the chromatic number of the Kneser graph $KG(C_2(n,k))$ is $n - 2k + 2$.

Generalizing this result, Meunier conjectured in 2011 that for all $n \geq sk$, the corresponding chromatic number for s-stable sets is $n - sk + s$. The conjecture was proved by P. Chen for even s, and by J. Jonsson for $s \geq 4$ and sufficiently large n. We prove the conjecture when $s = 3$ and n is sufficiently large, or when $k = s = 3$. To this end, we prove a version of the Hilton-Milner theorem for s-stable sets. We also present a topological approach toward Meunier’s conjecture.

Joint work with Wei-Chia Chen and Alex Parker.

Tuesday, July 21 · 11:00 a.m. · GHC 4405

Dual affine Robinson-Schensted correspondence via growth diagrams

Daoji Huang

ABSTRACT

The Robinson-Schensted (RS) correspondence admits diagrammatic interpretations via Fomin's growth diagrams and Viennot's shadow-line construction. Work of Spaltenstein, Springer, Steinberg, and van Leeuwen connected this combinatorial construction to the relative-position map of Springer fibers. Motivated by Kazhdan-Lusztig cell theory, various generalizations of the Robinson-Schensted correspondence to affine type A have been studied. Prominent examples include Shi's insertion algorithm and the affine matrix-ball construction.

In this talk, we introduce a new combinatorial construction of the affine RS correspondence via growth diagrams and shadow lines that is, in a sense, dual to Shi's insertion and the affine matrix-ball construction. We conjecture that the growth diagrams we construct admit a natural geometric realization in terms of relative positions of affine flags, similar to the interpretation given by Steinberg and van Leeuwen in the classical case.

Joint work with Sylvester Zhang.

Tuesday, July 21 · 11:30 a.m. · GHC 4405

A new ribbon basis for rank-selected homology of geometric lattices and representation stability

Patricia Hersh

ABSTRACT

We answer an old question of Anders Björner by providing a basis for the rank-selected homology of any geometric lattice. This basis, which we call the ribbon basis, is a matroid analogue of the polytabloid basis for a Specht module of ribbon shape. We prove that the action of Young symmetrizers on our ribbon basis satisfies a fundamental property of polytabloid bases. Using this feature of ribbon bases, we prove representation-theoretic stability for the rank-selected homology of both the Boolean lattice and the partition lattice, doing so in a way that gives sharp uniform representation-stability bounds in both cases. The sharp bound for the partition lattice proves a conjecture of Hersh and Reiner.

This is joint work with Sheila Sundaram.

Tuesday, July 21 · 12:00 p.m. · GHC 4405

The Gröbner version of White’s conjecture is false

Gaku Liu

ABSTRACT

We show that the toric ideal of the Fano matroid polytope does not have a quadratic Gröbner basis. This resolves in the negative a strong version of White’s conjecture from matroid theory. This result was found independently by De Loera, Ferroni, Morales, and Rambau.

Our approach is based on a new characterization of regular unimodular flag triangulations, which reduces the problem to an instance of SMT involving Boolean and real variables. We then use an SMT solver to prove unsatisfiability. Using this approach, we also show that all 8-element matroids that do not have the Fano matroid or its dual as a minor, with the possible exception of the matroid T_8, have toric ideals that admit quadratic Gröbner bases.

Tuesday, July 21 · 3:30 p.m. · GHC 4405

Existence of Kähler algebras with Chow polynomials as Hilbert series

Lorenzo Vecchi

ABSTRACT

In recent years, the introduction of Chow rings of matroids has led to breakthroughs in matroid theory, including proofs of combinatorial inequalities such as the Heron-Rota-Welsh and Mason conjectures.

Chow polynomials of posets generalize the Hilbert-Poincaré polynomial of the Chow ring of matroids. While no such ring-theoretic interpretation is known for general posets, several properties surprisingly still hold for the coefficients of this polynomial, including positivity, unimodality, and palindromicity. This observation has led to the conjecture that Chow rings should also generalize to arbitrary posets and, among other requirements, satisfy the Hard Lefschetz property.

As a step toward this conjecture, we show that the Chow polynomial of any poset satisfies the numerical constraints imposed by the Hilbert series of Hard Lefschetz algebras. Consequently, for any poset there exists an algebra with the Hard Lefschetz property-and even the full Kähler package-that has the Chow polynomial as its Hilbert-Poincaré polynomial. Equivalently, this shows that the coefficients of the Chow polynomial are the h-vector of a simple polytope.

This is joint work with Adam Schweitzer.

Tuesday, July 21 · 4:00 p.m. · GHC 4405

Principal minors of positive-definite matrices

Josephine Yu

ABSTRACT

Consider principal minors of positive-definite matrices. Is there a ratio of their products that is bounded for real symmetric positive-definite matrices but unbounded for complex Hermitian positive-definite matrices? I’ll discuss an answer involving tropical geometry, Grassmannians, matroids, and submodular functions.